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The Aggregation Problem: Implications for Ecological Economics

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Abstract

This article discusses the aggregation problem and its implications for ecological economics. The aggregation problem consists of a simple dilemma: when adding heterogeneous phenomena together, the observer must choose the unit of analysis. The dilemma is that this choice affects the resulting measurement. This means that aggregate measurements are dependent on one's goals, and on underlying theory. Using simple examples, this article shows how the aggregation problem complicates tasks such as calculating indexes of aggregate quantity, and how it undermines attempts to find a singular metric for complex issues such as sustainability.

Keywords: aggregation; GDP; capital stock; natural capital; sustainability indexes

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1 Introduction

Aggregation — the practice of summing heterogeneous things — is used in all aspects of science. Aggregation occurs when a physicist sums the mass of many different particles, or when an ecologist sums the energy consumption of an ecosystem. The use of aggregation is so commonplace that its epistemology is often given little thought. This is particularly true in economics — a field that tends to hide epistemological questions under a fog of mathematics (Mirowski, 1991; Keen, 2001). Aggregation is often portrayed as a purely objective process. After all what could be more objective than the simple arithmetic act of adding things up?

The purpose of this article is to show that aggregation, like any act of measurement, is never a purely objective process. This is because underlying every measurement are assumptions about what is to be quantified, and why the quantification is being made. Giampietro, Allen and Mayumi (2006) call this the “epistemological predicament associated with purposive quantitative analysis” — “the observer always affects what is observed when defining the descriptive domain”. This is another way of saying that no measurement is theory free.

In the natural sciences (particularly physics), the inter-connection between theory and observation is often undiscussed because there is a consensus that core theories are correct (and these theories are backed by overwhelming evidence). For instance, if a physicist wants to measure the inertia of a system, Newton’s laws make it clear that he/she should aggregate the system’s mass. But when we move into fields like ecological economics that study more complex phenomena, theory is less clear. For instance, what does it mean to measure ‘sustainability’? What is it that is to be sustained? And what are the units of measurement? If ecological economics is to be a “science of sustainability” (Dodds, 1997), then these epistemological questions must be addressed.

This article makes three main points. First, when it comes to aggregation and objectivity, *less is more*. When faced with theoretical uncertainties, less aggregation will generally lead to more objective analysis. Second, it is best to avoid aggregation that uses ‘real’ monetary value. The problem is that numerous subjective decisions must be made when attempting to ‘correct’ for inflation. Moreover, existing price-index methods (that underpin national accounting systems) are strongly informed by neoclassical economic theory. If one wishes to challenge neoclassical theory, then these methods should be avoided. Lastly, when we aggregate complex phenomena guided by theories that are either unclear or contested, we should acknowledge that the resulting measurement con-

tains an inescapable political element. This likely means abandoning the use of aggregation to search for ‘optimal’ policy decisions.

2 The Aggregation Problem

Any act of aggregation requires making two types of decisions. First, one must choose what is to be included in the aggregation and what is to be excluded. This is often called making *boundary decisions*. Second, one must choose a method for converting qualities into quantities. For simplicity, I call this choosing the ‘unit’ of analysis. Note that I mean this in the sense of choosing the *conceptual* unit (as in mass), not the *literal* measurement unit (as in kilograms or pounds).

This article focuses on the ‘unit’ aspect of aggregation analysis, rather than on boundary decisions. This is because boundary problems have already been extensively discussed in ecological economics literature. For instance, a common criticism of aggregate measures of output (i.e. real GDP) is that they do not include externalities such as environmental degradation or social ‘bads’ (Daly and Cobb, 1994; Kubiszewski et al., 2013). Similarly, ecological economists have criticized measures of the capital stock because they do not include the stock of natural resources, or ‘natural capital’ (Daly, 2011; Dixon and Hamilton, 1996; Costanza and Daly, 1992). While it is important to debate boundary decisions, my aim here is to show that even if there is a *consensus* on what system boundaries should be, the act of aggregation still involves subjective (theory-dependent) decisions about the unit of analysis. Moreover, these subjective decisions affect the measurement itself.

2.1 An Example: Aggregating Apples and Bread

The best way to understand how units affect the aggregation process is through a simple example. Suppose you are a shopkeeper who has a stock of apples and bread slices. Like many shopkeepers, you are not satisfied to state that you have x apples and y slices of bread. Instead, you want to know the size of your *total* inventory. How do you go about calculating this quantity?

Let’s set aside the fact that most shopkeepers care about the monetary value of their stock. (I will deal with monetary value later). Instead, let’s assume that you are a former natural scientist, and you want a physical measure of the size of your stock. This is simple enough to do — all that is required is for you to choose a unit of analysis. Table 1 shows realistic values for the average mass, volume and energy content of apples and bread slices. You simply choose one of

these units, and use it to aggregate your total stock. But herein lies the problem. The choice of units is subjective — it depends on your goals. Yet this choice plays a crucial role in determining the measurement results.

Table 1: Measuring apples and bread slices using different units

| | Mass (g) | Volume (cm ³) | Energy (cal) |
|-------------|----------|---------------------------|--------------|
| Apple | 75 | 104 | 39 |
| Bread Slice | 30 | 52 | 79 |

To understand this dilemma, it is helpful to reflect on what a unit does. In an aggregative analysis, a unit determines the relative weights assigned to the different elements being added together. In our example, the unit determines how we weight apples relative to bread slices. The problem is that different units lead to different weightings. Using the values in Table 1, we can see that using mass, volume, or energy leads to the following (different) weightings between apples and bread slices:

$$\text{Mass: } 1 \text{ apple} = 2.5 \text{ bread slices} \quad (1)$$

$$\text{Volume: } 1 \text{ apple} = 2.0 \text{ bread slices} \quad (2)$$

$$\text{Energy: } 1 \text{ apple} = 0.5 \text{ bread slices} \quad (3)$$

These different weightings can lead to wildly divergent measures for the aggregate stock of apples and bread slices. A clear way to illustrate the problem is to construct an indexed time series of aggregate quantity (an extremely common practice in economics). Suppose that over the course of 30 hours, the individual stock of apples and bread slices changes as shown in Figure 1A. Assuming that apples and bread slices are uniform, we can objectively state that the stock of bread slices increases by 164%, while the stock of apples decreases by 70%. There is no ambiguity here. We would get the same result, no matter what unit of analysis we choose.

However, this is *not* true when we move to an aggregate analysis. Figure 1B shows the results of aggregating the stock of apples and bread slices using units of energy, volume, and mass (with values from Table 1). Suddenly there is significant ambiguity in the indexed growth of the aggregate stock. When measured in terms of caloric energy, the size of our apple-bread stock increases by 86%. Yet when measured in terms of mass, the same stock appears to decrease in size by 3%. This large discrepancy occurs because when we change units, we

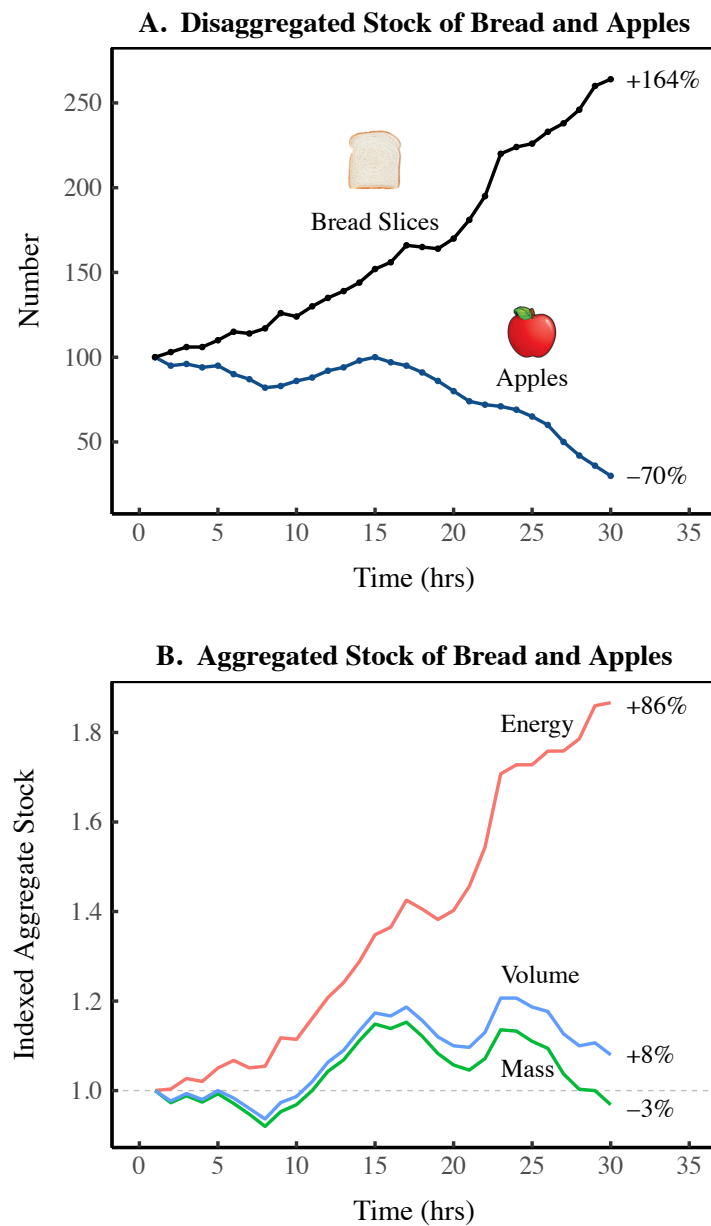


Figure 1: Conflicting aggregate measures of a stock of apples and bread slices

This figure shows how the choice of (conceptual) unit affects aggregate measures of quantity. We imagine that a shopkeeper has a stock of apples and bread slices. Panel A shows how the number of apples and bread slices changes over a period of 30 hours. Panel B shows three different indexed aggregate measures of the same stock, calculated using units of energy, volume, and mass (with values from Table 1). Different units lead to a different weighting between apples and bread slices, which causes divergent measures for the growth of the aggregate stock. Notes: this figure is inspired by Fig. 8.1 in [Nitzan and Bichler \(2009\)](#).

change the relative weighting between apples and bread slices. This, in turn, affects how much we weight the *increase* in the quantity of bread slices against the *decrease* in the quantity of apples. Because the different units yield different apple-to-bread-slice weightings, the resulting indexes of aggregate quantity give conflicting results.

It might seem reasonable to ask — which of these three indexes is the ‘correct’ measure of aggregate quantity? However this question is ill posed. All three measures are correct in a strictly mathematical sense. Instead, what we should ask is — which measurement is *appropriate*, given our goals and our state of knowledge? It is here that subjectivity enters the equation. For instance, if we want to know the scale of our apple-bread stock in the context of feeding a starving population, caloric energy content seems the most appropriate choice of unit. But if we wanted to calculate shipping costs, then mass is likely the best unit. Even when the context makes the choice of the unit seem clear, we need to remember that the aggregation process depends on a priori decisions about why we are taking the measurement.

The subjective aspect of aggregation means that in matters where the appropriate unit is not clear (or when the unit is contested), aggregate analysis should be undertaken with caution. For instance, suppose that instead of aggregating bread and apples, we wanted to aggregate fresh water and bituminous coal to create an index of ‘natural capital’. It is far from clear what unit of analysis we should use, since fresh water and bituminous coal have completely different uses. Given the uncertainty in our goals and theory, the resulting aggregation would have a great deal of ambiguity. Thus, it is far more reasonable to treat fresh water and bituminous coal as separate, incommensurable entities. This disaggregated treatment will be far more objective than any aggregate analysis.

To summarize, aggregation always involves theory-informed choices about the unit of analysis, and these choices affect the resulting measurement. Given this epistemological predicament, researchers need to remember that less aggregation means greater objectivity.

3 Monetary Value: The Changing Meter Stick

A defining feature of economics is its focus on prices. This has led to a strong tendency to conduct aggregate analysis using units of monetary value. Unfortunately, using prices as the unit of analysis leads to its own unique set of problems. The difficulty is that prices *change over time*, and attempts to ‘adjust’ for this change inevitably require subjective decisions.

When we use prices to measure how aggregate quantities (such as economic output) change over time, a common belief is that one can objectively account for price changes simply by adjusting for inflation (using official price indexes). However, the matter is not so simple. The problem is that price changes are not uniform. As shown in Figure 2A, historical price changes (in the US) have varied drastically by commodity. Since 1935, the price of apples increased by a factor of 50, the price of electricity increased by a factor of 7, and the price of TVs actually declined (more on this later). This divergent price change means that our unit is *unstable*. The effect is the same as when we literally changed units in our apple-bread example (Fig. 1). Divergent price changes cause the relative weighting between commodities to change with time. This means that our aggregate measure will be affected by the year in which we chose our prices. This problem was identified over a century ago by Francis Edgeworth (Edgeworth, 1887):

If one great group of commodities varies pretty uniformly in one direction, and another in a different direction (or even in the same direction but in a markedly different degree), then the task of restoring the level of prices can no longer be regarded as a *purely objective* ... problem. (cited in Vining and Elwertowski (1976); emphasis added)

The effect of changing prices can be clearly illustrated by calculating real GDP using different base years (in which prices are fixed). Figure 2B shows how the choice of base year affects the growth of US real GDP. This analysis indicates a 30% uncertainty in the growth of US GDP over the last 60 years.

This range of estimates is conservative because it does not account for other subjective factors that enter into price adjustments. Most importantly, commodities themselves change with time. Today's computers are drastically different from those of the 1990s. It is standard practice, in price-index methodology, to differentiate between pure changes in price (inflation) and changes in the *quality* of a commodity. For instance, if a computer increases in price by a factor of 2, but at the same time increases in 'quality' by a factor of 4, this is recorded as a *decrease* in price by a factor of 2. This is why Figure 2A shows such a drastic decrease in the price of computers. Virtually all of it is due to quality adjustments. The same is true of TVs (which have drastically decreased in indexed price since the advent of smart TVs).

How is this change in quality measured? Here the aggregation problem rears its head again. In order to measure quality change, we must decompose a commodity into individual components, measure the change in quality of these components, and then aggregate the result. But what unit should we chose? Non-

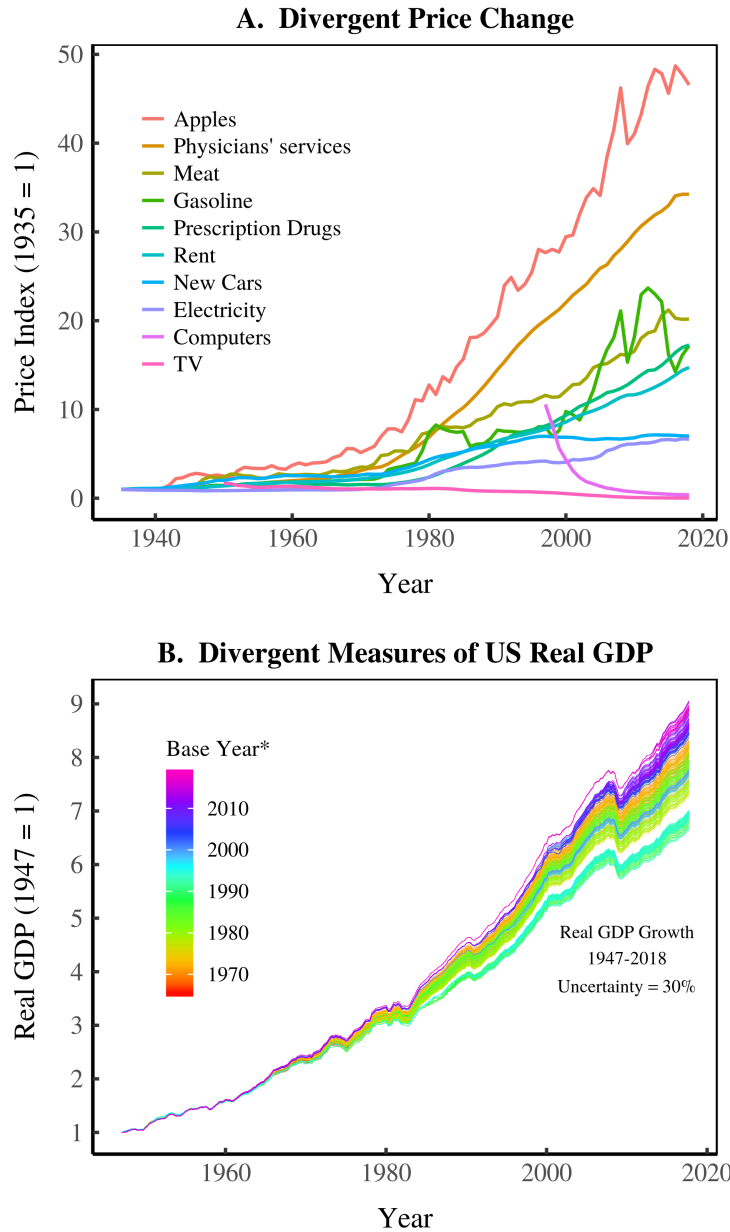


Figure 2: Divergent Price Change

This figure shows how divergent changes in price affect the measurement of US real GDP. Panel A shows historical price changes in ten selected commodities tracked by the Bureau of Labor Statistics. Divergent price change means that the choice of base year has a strong effect on the measurement of real GDP growth, as shown in Panel B. Methods: in Panel A, some commodities are introduced after 1935 (such as TVs and computers). For comparison, I index the price of these new commodities to the mean of the other commodities in the year of introduction. Data for Panel B comes from the Federal Reserve Bank of Philadelphia, series routputmvqd. Note that the source data contains real GDP calculations only for years prior to each series' corresponding base year. For comparison, I project real GDP growth up to 2017 (for all series). I do this using growth rates for the 2017 series, less historical differences in mean growth rates between the 2017 series and the vintage series in question.

specialists in price-index methodology may be surprised that statistical agencies use units of *utility* — the pleasure/satisfaction derived from a product. For instance, when describing the ‘hedonic’ methods used to differentiate changes in a commodity’s price from changes in quality, the US Bureau of Labor Statistics writes:

In price index methodology, hedonic quality adjustment has come to mean the practice of decomposing an item into its constituent characteristics, obtaining estimates of the *value of the utility* derived from each characteristic, and using those value estimates to adjust prices when the quality of a good changes. (BLS, 2010) [emphasis added]

This utilitarian approach is problematic. As Joan Robinson (1962) famously observed, utility is a circular concept. “*Utility*”, Robinson writes, “ is the quality in commodities that makes individuals want to buy them, and the fact that individuals want to buy commodities shows that they have *utility*” (emphasis in original). When we aggregate in units of utility, not only is the choice of unit subjective, but the very unit itself is impossible to observe (Nitzan and Bichler, 2009). Furthermore, the utilitarian theory of value is an explicitly *neoclassical* construct. If ecological economists wish to contest neoclassical theory, then they need to be aware that existing methods for calculating ‘real’ monetary value are strongly neoclassical.

I have focused exclusively on an aggregation example (real GDP) where exact market prices are known. My point has been to show that, even when there is no ambiguity in market value, there is no way to use prices to uncover an objective ‘real’ quantity. The aggregate analysis still requires subjective, theory-informed choices that affect the resulting measurement. When we move, as ecological economists do, to attempting to aggregate the value of things that have no market price (such as natural capital or social/environmental externalities), the problem is even more severe. Often ecological economists use neoclassical valuation methods (such as contingent valuation) without a discussion of the underlying problems, and without investigating how alternative methods would change the results. For a discussion of the problems with neoclassical valuation, see Diamond and Hausman (1994), Dore (1996), and Eberle and Hayden (1991).

For those interested in the many problems with using prices to uncover ‘real’ quantities, a good starting point is to revisit the Cambridge capital controversy. This was a debate in the 1950s and 1960s between economists in Cambridge, England and Cambridge, Massachusetts over how capital was to be measured. Joan Robinson (1953) began the debate when she asked — in what units is

capital measured? This prompted a protracted exchange that culminating in the Cambridge, England school demonstrating that there is no way to measure the quantity of capital independently of prices and distribution ([Hodgson, 2005](#)).

This finding has important implications for ecological economists who wish to value non-market phenomena, because it means that any such valuation will depend on existing prices and the existing distribution of income. If prices (or the distribution of income) change, then the valuation will change. (For those interested, [Cohen and Harcourt \(2003\)](#), [Felipe and Fisher \(2003\)](#), and [Harcourt \(2015\)](#) provide a good summary of the Cambridge debate. For a broader discussion of the problems with measuring capital see [Nitzan and Bichler \(2009\)](#). For an extensive review of problems with differentiating between price change and quantity change, see [Nitzan \(1992\)](#)).

4 The Political Element

When dealing with complex phenomena described by vague or contested theory, there is typically a political element to aggregation, if only in the choice of unit. Perhaps the best example of a political aggregation decision is the method for counting state population that was stipulated in the original US constitution. Early on in the 1787 Constitutional Convention, it was decided that Federal representation would be determined by state population. But what was the *unit* of population? The infamous ‘Three-Fifths Compromise’ dictated that the unit would be ‘free persons’, and that slaves would count as three fifths of a free person. Since slaves could not vote, but were nonetheless counted towards population, the effect was to give free individuals in slave-owning states greater control over government than their northern counterparts. This allowed Southern states to dominate Federal politics during the pre-Civil War era ([Addison, 2009](#); [Wills, 2005](#)).

While few examples are as overtly political as this, when we aggregate complex phenomena, the underlying measurement decisions often have political consequences. For instance, the choice of unit can affect how we view the ‘environmental impact’ of a technology. Consider recent debates over whether the diesel engine is more environmentally friendly than the gasoline engine. If ‘environmental impact’ is measured in terms of carbon emissions, then diesel is the superior technology. (Diesel engines are more fuel efficient, and therefore emit less carbon dioxide). However, if ‘environmental impact’ is to be measured in terms of health-affecting pollutants such as particulate matter or nitrogen oxides, then diesel engines are worse than gasoline engines ([Ghose, 2015](#)). Unfor-

tunately, this is no thought experiment. In an effort to meet Kyoto obligations, many European countries promoted a rapid switch from gasoline to diesel cars without a regard for how this would affect air quality (Forrest, 2017). In this case, the choice of units meant deciding between “killing people today rather than saving lives tomorrow” (Vidal, 2015).

Environmental decision-making is often characterized by this type of messy trade-off that has no easy solution. What is troubling is not that measurement choices have political consequences, but rather, the tendency in economics to use aggregation to seek *optimal* solutions to complex problems. Thus economists have theories for (among other things) optimal taxation (Sandmo, 1976), optimal government size (Karras, 1996), optimal economic growth (Koopmans, 1965), and optimal levels of pollution control (Kwerel, 1977). This type of theory has the effect of shutting down debate. After all, it is hard to argue with a policy that is ‘optimal’.

The problem is that so-called ‘optimal’ solutions are always predicated on subjective decisions about what is to be optimized (and how it is to be measured). Without decisions about one’s objective, multidimensional decisions do not have optimums, they have *trade-offs*. This can be illustrated by using our example of a stock of apples and bread slices. Suppose we need to maximize our stock by choosing between two scenarios. In Scenario A, we have 3 apples and 2 bread slices, while in Scenario B, we have 2 apples and 3 bread slices. Which stock is larger? Without further defining our objective, this question has no meaning. Scenario A and Scenario B involve a *trade-off* between an additional apple or an additional bread slice. Since these scenarios are qualitatively different, we cannot make a judgment about which situation maximizes the stock.

In order to make such a judgment, we must aggregate the apple-bread stock into a single metric. Unfortunately, the choice of aggregation unit affects what we find. As shown in Figure 3, when we aggregate in units of energy, we find that Scenario A maximizes the apple-bread stock. However, when we aggregate in units of mass, Scenario B maximizes the stock. Again, this is because the unit affects the relative weighting between apples and bread slices. As this simple example illustrates, the appearance of optimality is determined by subjective decisions used during aggregation. Different decisions will lead to different ‘optimal’ solutions.

In a more complex example, suppose when choosing between diesel and gasoline engines, we use units of lives lost to aggregate the effects of carbon emissions with the effects of particulate matter. This type of analysis necessarily involves numerous subjective decisions. For instance, how should we weight

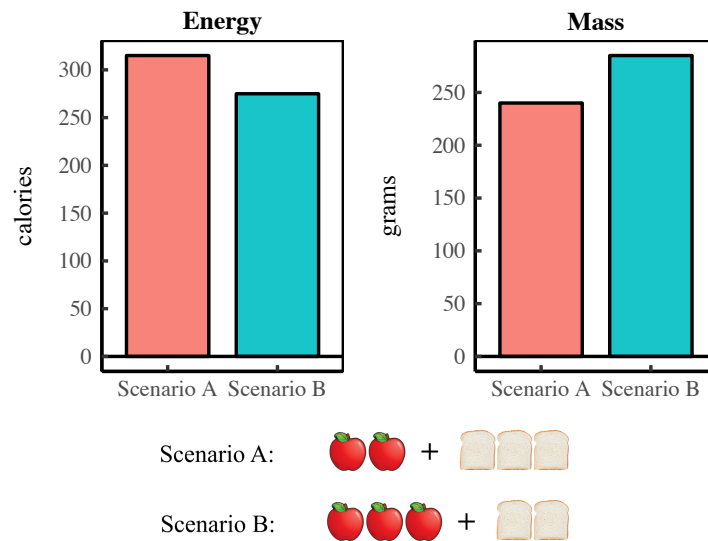


Figure 3: Conflicting measurements for the maximum stock of apples and bread slices

This figure shows how the choice of unit affects the choice between two different scenarios (A and B). Which scenario maximizes the apple-bread stock? Our finding depends on the unit of analysis. When measured in terms of caloric energy (left), Scenario A maximizes the stock. However, when measured in terms of mass, Scenario B maximizes the stock. Calculations use values from Table 1.

the immediate (local) deaths from poor air quality against the future (global) deaths from climate change. And how should we convert chronic illness caused by pollution into units of lives lost? To convert these heterogeneous qualities into a single quantity, we must make subjective decisions. Importantly, these decisions will affect the ‘optimal’ balance between carbon and particulate-matter emissions.

In my view, seeking ‘optimum’ solutions to complex problems via cost-benefit analysis is a pernicious use of aggregation. It has the effect of obscuring subjective trade-offs that are otherwise clearly visible in a disaggregated analysis. This transforms a political debate into a technical dispute accessible to ‘experts’ only. But as [Ackerman \(2008\)](#) observes, the problem is not with collecting quantitative data, it is the last step of aggregating everything together:

Most of the information collected for a cost-benefit analysis is useful under any approach to deliberation. The problems arise only in the final steps of crunching everything into a single bottom-line number: monetizing nonmonetary benefits, discounting future outcomes, and guesstimating the values of important uncertainties all have the effect of distorting and concealing the

underlying data.

Thus one possible alternative to cost-benefit analysis is to simply leave information about environmental impact in disaggregated form, with each aspect expressed in its ‘natural units’ (i.e. habitat loss in units of area, pollution in units of mass, lives lost units of individuals, etc.). Stakeholders could interpret this information as they see fit in order to make a judgment. This would maintain a clear separation between objective quantitative analysis and subjective decision-making.

5 Conclusion

The crux of the aggregation problem is this: when we convert qualitatively different phenomena into a single, aggregate quantity, the qualitative differences do not just disappear. Instead, these differences become *hidden* as unexpressed information. For any given aggregation, what we do not see are the many other possible ways of converting qualities into quantities. While the aggregation problem is simple when it is identified, it is often overlooked. Why? A plausible reason is that when we deal with different phenomena (apples and oranges) in qualitative terms, most people have an intuitive understanding that a wide variety of comparisons is possible. But when we convert to a common (quantitative) unit, it takes a concerted effort to see how the choice of unit affects the analysis.

None of this is to say that aggregation should always be avoided. Without aggregation, quantitative science would be impossible. Instead, I am suggesting that the usefulness of an aggregate metric should be judged in relation to the accuracy of the theories on which it is based. In disciplines like physics where core theories are overwhelmingly confirmed by evidence (and units are meticulously defined), then aggregation is unproblematic. However, in fields like ecological economics that deal with complex issues at the forefront of human knowledge, the matter is different. Here, aggregation may do more harm than good because underlying theories are often vague, highly contested, and units are poorly defined. The effect may be to give the illusion of quantitative certainty when there is none. In particular, the search for a single metric of sustainability or a single metric for environmental impact is likely a fools errand (and it is particularly pernicious if this metric uses units of monetary value).

If ecological economics is to be a ‘science of sustainability’, then it must take the aggregation problem seriously. This may mean lowering our expectations of what can be measured with pure objectivity, and being forthright about the subjective elements in our aggregate metrics. Alternatively, we can simply assess

environmental impacts in a disaggregated manner in order to separate objective measurement from subjective decision-making.

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